**Logo

Description automatically generated San Francisco Bay University**

**MATH201 - Calculus-I**

**Homework Assignment #3**

**Naim Rahman Ifti**

**Id: 20037**

**Due day: 10/21/2024**

**Instruction:**

1. **Push the answer sheet to Github in word file**
2. **Overdue homework submission could not be accepted.**
3. **Takes academic honesty and integrity seriously (Zero Tolerance of Cheating & Plagiarism)**
4. **If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters *t* seconds later is given by**

**(a) Find the average velocity over the given time intervals:**

**(i) [1, 2] (ii) [1, 1.5] (iii) [1, 1.1]**

**(iv) [1, 1.01] (v) [1, 1.001]**

**Answer:**

The height of a rock thrown upward on Mars is given by the equation:

y= 10t − 1.86t2

where y is the height in meters, and t is the time in seconds.

To find the average velocity over an interval [t1, t2] use the formula for average velocity:

A black text on a white background

Description automatically generated

Now, let's calculate y(t1) and y(t2) for each time interval:

1. Interval: [1, 2]

A number equation with numbers and lines

Description automatically generated with medium confidence

1. Interval: [1, 1.5]

A number with numbers and lines

Description automatically generated with medium confidence

1. Interval: [1, 1.1]

A number with numbers and lines

Description automatically generated with medium confidence

1. Interval: [1, 1.01]

A number with numbers and lines

Description automatically generated with medium confidence

1. Interval: [1, 1.001]

A number with numbers and lines

Description automatically generated with medium confidence

**(b) Estimate the instantaneous velocity in Excel when**

**Answer:**

Given,

Y = 10t − 1.86t2

To find the instantaneous velocity at t=1, we need to take the derivative of the height function with respect to time and then evaluate it at t=1.

The derivative of the height equation y with respect to time t is:

vy​(t) =10 − 3.72t

Evaluating this at t=1 gives the instantaneous velocity:

vy​(1) = 6.28m/s

A screenshot of a computer

Description automatically generated

1. **The displacement (in centimeters) of a particle moving back and forth along a straight line is given by the equation of motion , where *t* is measured in seconds.**

**(a) Find the average velocity during each time period:**

**(i) [1, 2] (ii) [1, 1.1]**

**(iii) [1, 1.01] (iv) [1, 1.001]**

**Answer:**

Given,

s = 2sin(πt) + 3cos(πt)

The average velocity Vavg​ over a time interval [t1,t2] is given by:

**A black and white math equation

Description automatically generated**

1. Interval [1,2]:

A math equations with numbers and symbols

Description automatically generated

1. Interval [1,1.1]:



A number and number equation

Description automatically generated with medium confidence

1. Interval [1,1.01]:

A black text with a x and a cross

Description automatically generated

A mathematical equation with numbers and symbols

Description automatically generated

1. Interval [1,1.001]:

A black text on a white background

Description automatically generated

A number equation with numbers and lines

Description automatically generated with medium confidence

**(b) Estimate the instantaneous velocity of the particle in Excel when**

**Answer:**

To estimate the instantaneous velocity at t=1, we need to calculate the slope of the secant lines for increasingly smaller time intervals around t=1. This will involve computing the average velocity over the intervals, which is given by:

**A black and white math equation

Description automatically generated**

Where:

* s(t) is the displacement function.
* t1 is 1, and t2​ are the values slightly greater than 1.

A screenshot of a calculator

Description automatically generated

To estimate the instantaneous velocity at t=1, we can see that the values for smaller intervals approach −6.27 cm/s.

1. **(a) Estimate the value of**

**by graphing the function in Excel. State your answer correct to two decimal places.**

**Answer:**

To estimate the value of limx→0​sin(πx)sin(x)​ by graphing the function f(x)=sin(πx)sin(x)​ in Excel:

**A screenshot of a spreadsheet

Description automatically generated**

Let's graph the function for xxx values very close to zero to get an estimate of the limit.

A graph of function with numbers

Description automatically generated

The graph of the function f(x)=sin(πx)sin(x)​ near x=0 appears to approach a constant value as x approaches 0. From the plot, it looks like the function stabilizes at 1, suggesting that:

A mathematical equation with numbers and symbols

Description automatically generated

**(b) Check your answer in part (a) by evaluating for values of *x* that approaches 0 in Excel.**

**Answer:**

The function f(x)= sin(x) / sin(πx)​ measures the ratio of the sine of x to the sine of πx. As x approaches 0, both the numerator and denominator approach 0, which is an indeterminate form 0/0​.

A known mathematical limit that may help here is: 1limx→0​ sin(kx)​ / kx =1 for any constant k. Applying this to both the numerator and the denominator:

**A number and number symbols

Description automatically generated with medium confidence**

Let's choose values of x close to 0 (like 0.1,0.01,0.001 etc.) and evaluate f(x) to check the consistency of our result with the expected 1/π ​≈0.318:

* f(0.1)
* f(0.01)
* f(0.001)
* f(0.0001)

These results are consistently close to 1/π ​≈0.318, confirming our expected analytical result. As xxx gets closer to 0, f(x) approaches approximately 0.318, supporting the hypothesis from the analytical derivation that

****

**In Excel:**

**A screenshot of a computer

Description automatically generated**

**A graph on a white sheet

Description automatically generated**

1. **(a) Estimate the value of the limit to five decimal places. Does this number look familiar?**

**Answer:**

start by taking the natural logarithm of the expression (1+x)1/x :

**A math equation with numbers and symbols

Description automatically generated**

The Taylor series expansion of ln(1+x around x= 0 is:

**A mathematical equation with numbers and symbols

Description automatically generated**

Substitute this back into the transformed expression:

**A mathematical equation with numbers

Description automatically generated**

**A black and white math equation

Description automatically generated**

**A black text on a white background

Description automatically generated**

By taking the exponential of both sides:

A math equations with numbers and symbols

Description automatically generated with medium confidence

This shows that limx→0(1+x)1/x equals e, and this result is foundational in calculus, particularly in defining the natural exponential function ex.

Yes, the number is familiar.

**(b) Illustrate part (a) by graphing the function in Excel**

**Answer:**

**In excel:**

A graph of a graph

Description automatically generated

1. **(a) Graph the function for in Excel. Do you think the graph is an accurate representation of *f*?**

**Answer:**

**The excel sheet is:**

**A screenshot of a computer

Description automatically generated**

**Graph:**

**A graph of a function

Description automatically generated**

**(b) How would you get a graph that represents *f* better?**

**Answer:**

To provide a mathematical understanding of the behavior of the function |f(x)=ex+ln∣x−4∣ around the point x=4, and to justify the need for special attention (like a vertical asymptote) at this point, we can consider both the components of the function separately and then combined.

### Mathematical Examination:

1. **Exponential Part (ex)**:
   * The function ex is defined for all real numbers x and is continuous and smooth everywhere. It grows rapidly as x increases.
2. **Logarithmic Part (ln∣x−4∣)**:
   * The logarithm of a number is defined only for positive arguments. Hence, ln∣x−4∣ is undefined at x=4 because ∣4−4∣=0, and the logarithm of zero is undefined.
   * As x approaches 4 from either side, x−4 approaches zero, and the logarithm approaches −∞. Mathematically:

A number of equations on a white background

Description automatically generated

* + This implies that there should be a vertical asymptote at x=4, as ln∣x−4∣ causes the function to shoot down to negative infinity.

### Combining the Parts:

* The overall function f(x)= ex + ln∣x−4∣ is therefore characterized by the rapid growth of ex and the steep decline of ln∣x−4∣ near x=4. The exponential growth of ex cannot counterbalance the logarithmic term diving to negative infinity as x approaches 4.
* Consequently, despite the exponential term, the function will exhibit a behavior where it tends to negative infinity as x approaches 4 from either side, reaffirming the existence of a vertical asymptote at x = 4.

### Graphical Representation:

To graph this function accurately, especially around x = 4, capturing these behaviors is crucial. As suggested earlier, employing finer intervals around this point would help illustrate the rapid change more clearly.

1. **(a) Use numerical to find the value of the limit and verify it in Excel**

**Answer:**

**By using the numerical value the value of limit is:**

**A screenshot of a computer

Description automatically generated**

**(b) How close to *1* does *x* have to be to ensure that the function in part (a) is within a distance *0.5* of its limit?**

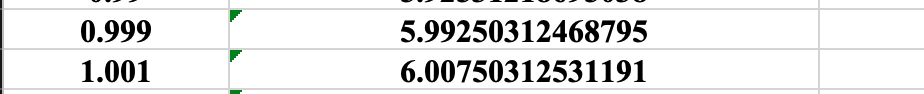
**Answer:**

From the excel file in part (a),  
The limit as x approaches 1 from either side is evaluated using the expression

A close-up of a number

Description automatically generated

The values in the table near x=1 are:



Assuming the limit value L is approximately 6 (as the values are tending towards this number as xxx approaches 1), we need to determine how close x needs to be to 1 to ensure ∣f(x)−L∣ < 0.5.

From the provided values:

* At x=0.999, f(x)=5.99250312468795 which is approximately 6−0.007496875312056
* At x=1.001, f(x) = 6.00750312531191 which is approximately 6 + 0.007503125311916

Since these are very close to 6 and well within 0.5 of the limit, we conclude:

* The values at x=0.999 and x = 1.001 already satisfy the condition ∣f(x)−6∣<0.5.

Thus, to ensure the function value is within 0.5 of the limit (approximately 6):

* x needs to be within the interval [0.999,1.001] around 1.

This means x should be within 0.001 of 1 to maintain the function value within 0.5 of the limit.